Math 102

Krishanu Sankar

November 29, 2018

Announcements

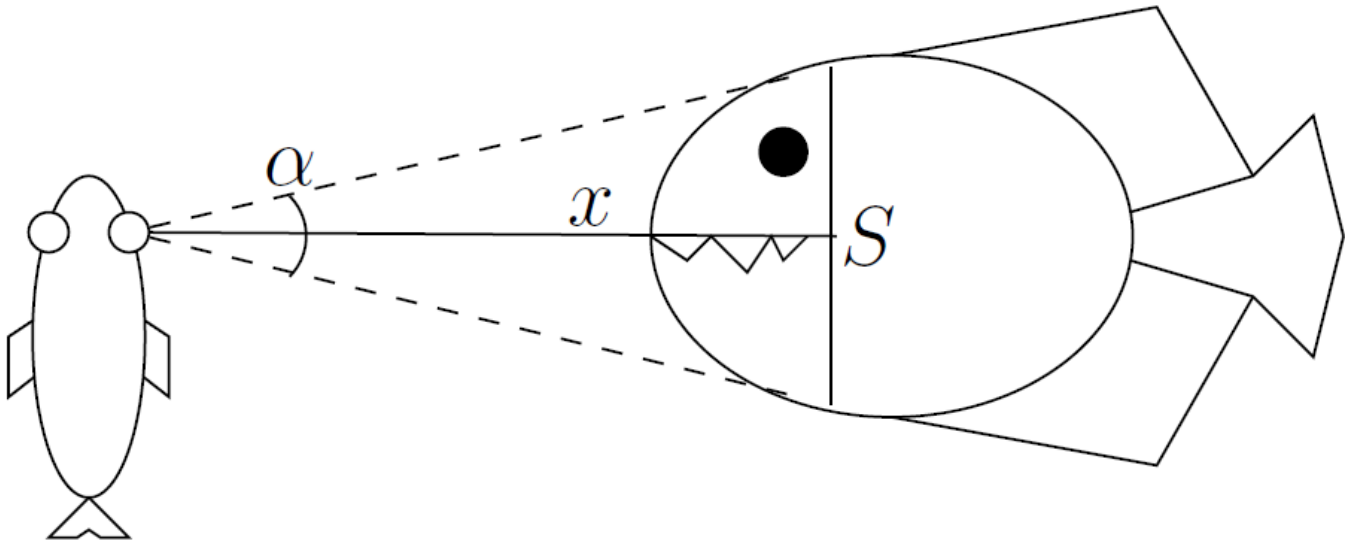
- Review sessions
  - Today and tomorrow 4-7PM, in SWNG 122.
  - Mine is today (Thursday) 6-7PM. I plan to go through the 2013 WT1 Final Exam and work through questions selected by popular demand.
- Final Exam Information
  - **When:** Tuesday Dec 4, 3:30 to 6:00 PM.
  - **Where:** Math 100.
  - No calculators, no notes. Bring your UBC ID.
  - Format is similar to the midterm, but 2.5 hours.
- Course Evaluations
  - Accessible from Canvas (see Course Evaluations tab on left)
  - Close on December 3
  - Should take about 10-15 minutes
  - Please complete it! The feedback is valuable to me and the department.

Goals Today

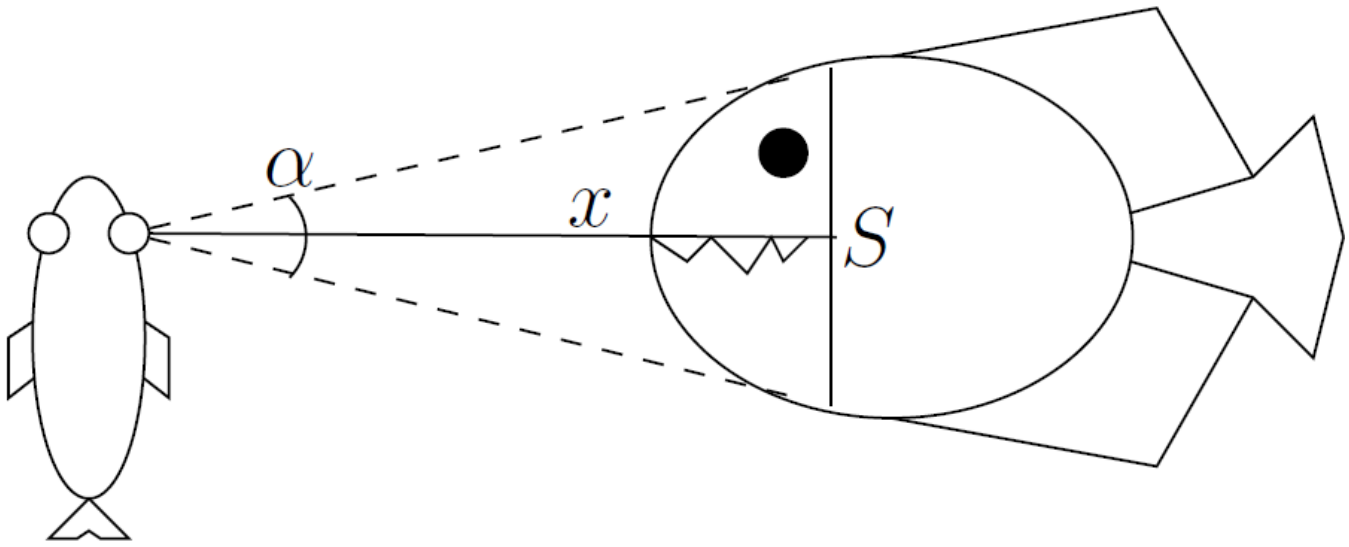
- Related rates - Zebrafish predator response

The Escape Response

Zebrafish flee if they determine that a predator is approaching. Larry Dill, a biologist at Simon Fraser University, hypothesized that zebrafish's escape response is triggered when the *visual angle* subtended by an approaching object changes rapidly enough. That is, when $\frac{d\alpha}{dt} > K_{\text{crit}}$, for some critical threshold K_{crit}.

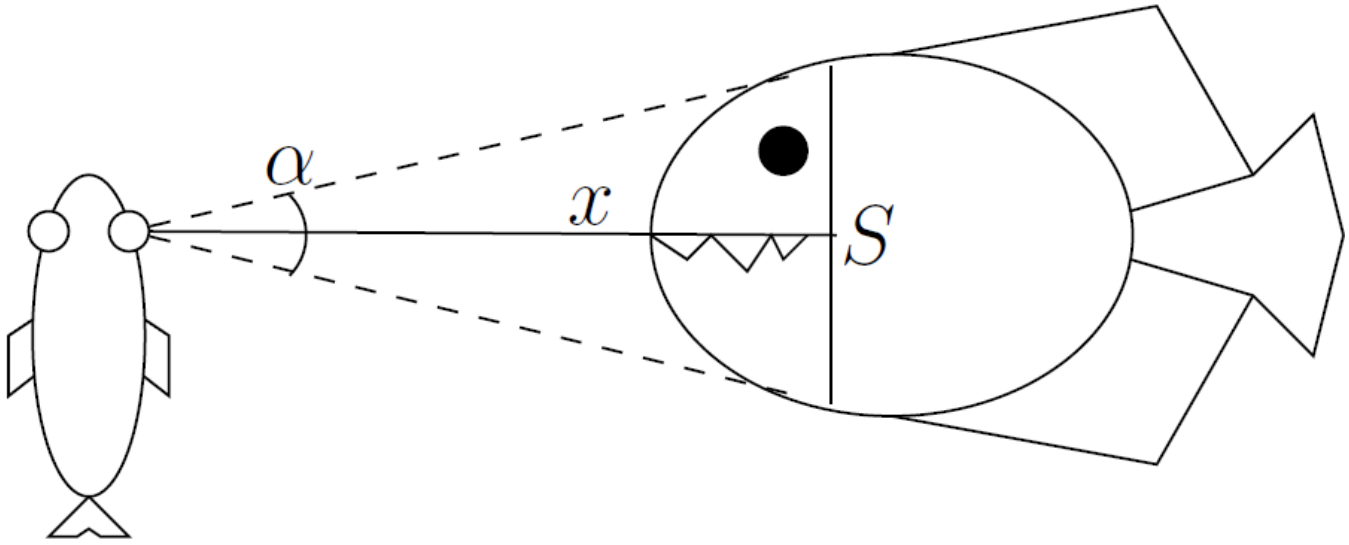


The Escape Response

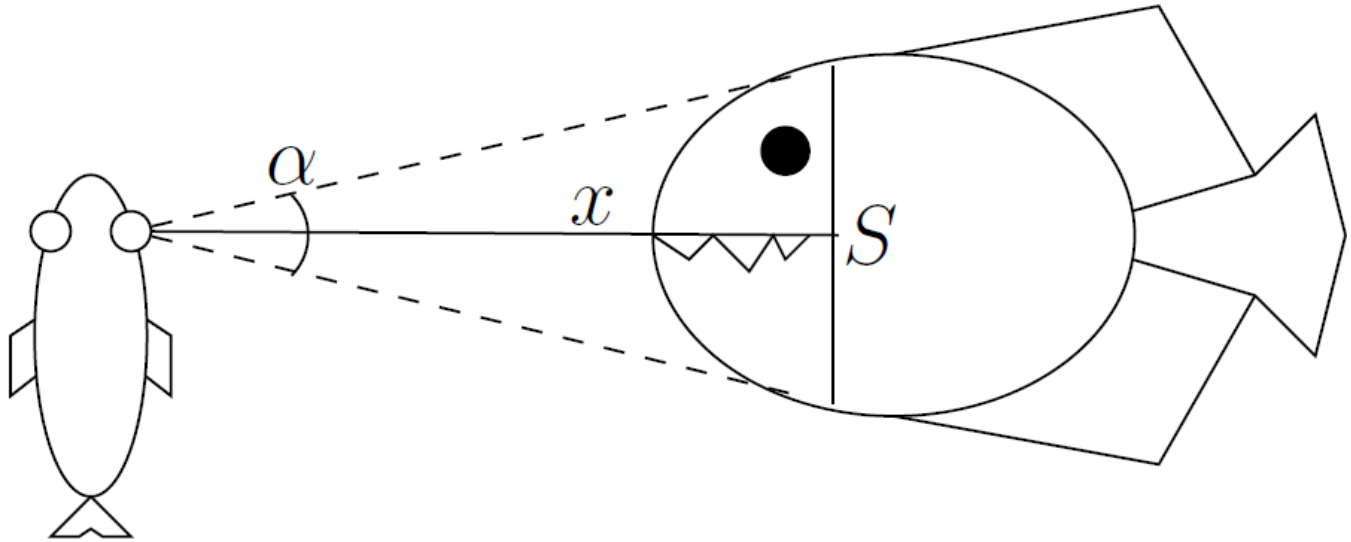


- $x =$ distance from zebrafish to predator
- $S =$ size of predator
- $\alpha =$ visual angle
- $K_{\text{crit}} =$ constant

When is $\frac{d\alpha}{dt} > K_{\text{crit}}$?



Question: Suppose that the zebrafish approaches at a constant speed of v. That is, $\frac{dx}{dt} = -v$. Then calculate $\frac{d\alpha}{dt}$ as a function of x, S, and v.



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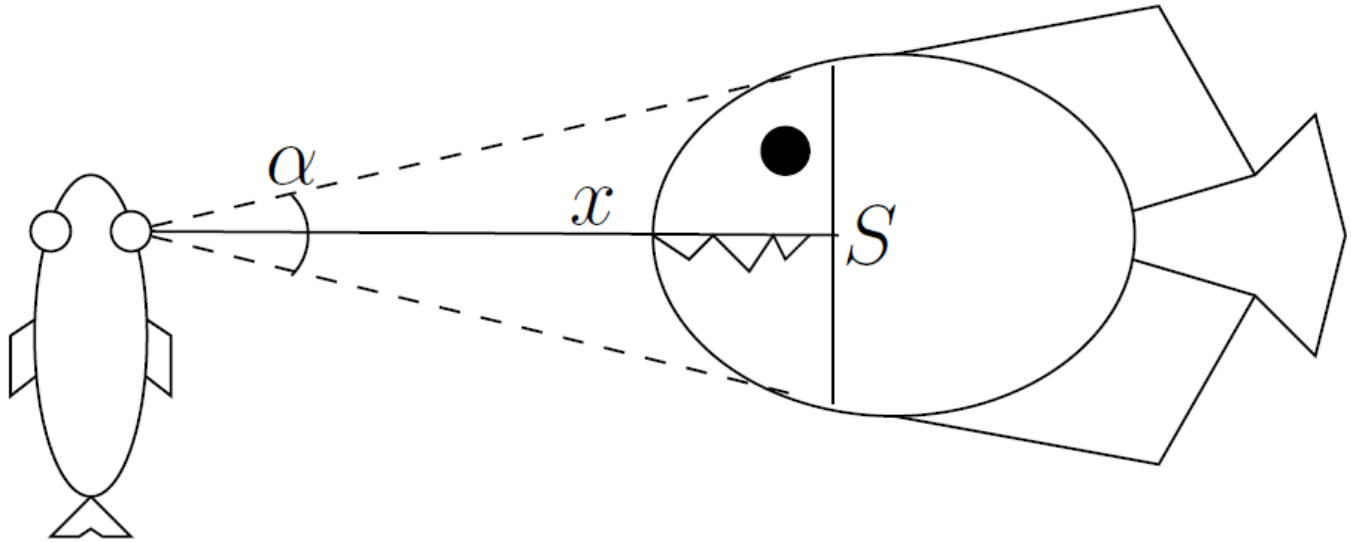
$$\alpha = 2 \arctan\left(\frac{S}{2x}\right)$$

$$\frac{d\alpha}{dt} = \frac{2}{1 + (S/2x)^2} \frac{d(S/2x)}{dt}$$



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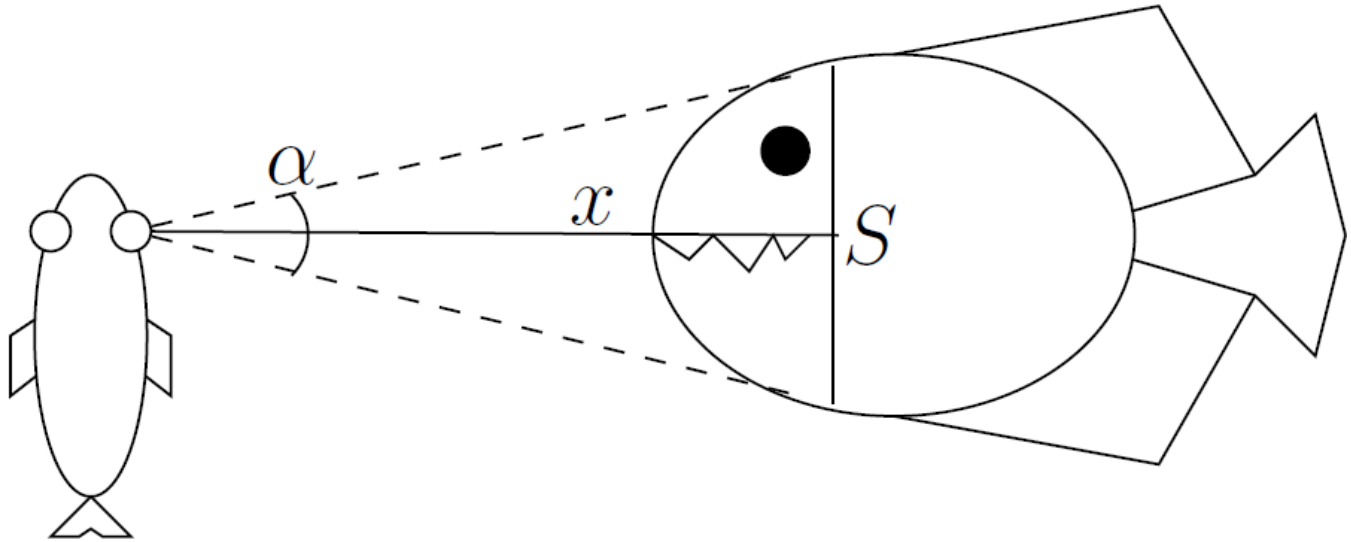

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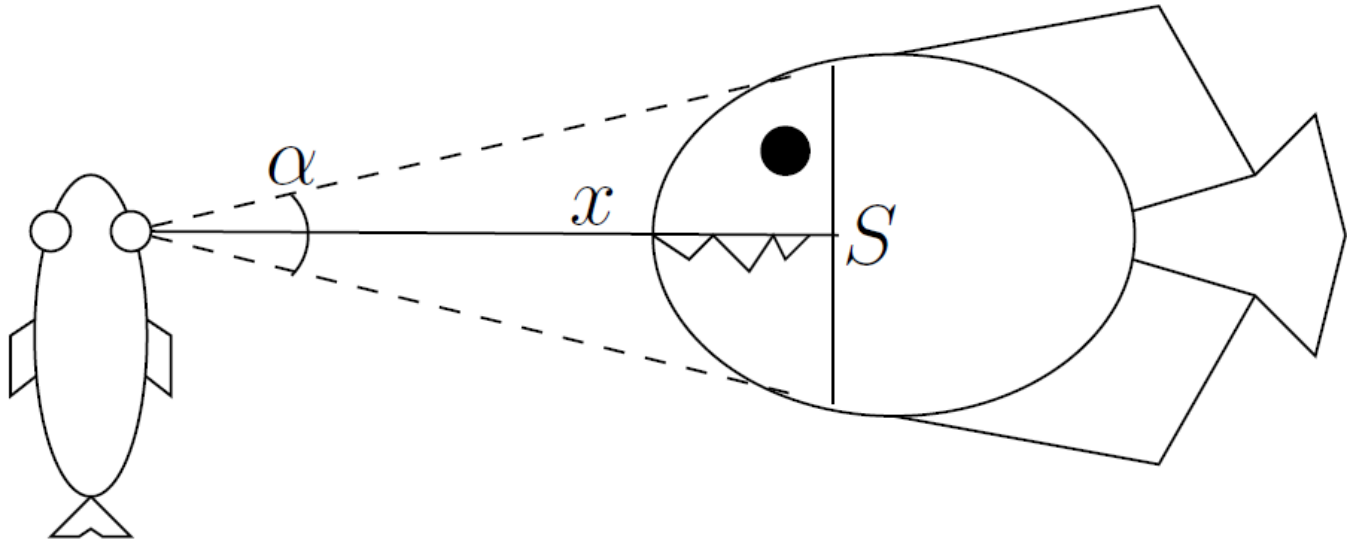

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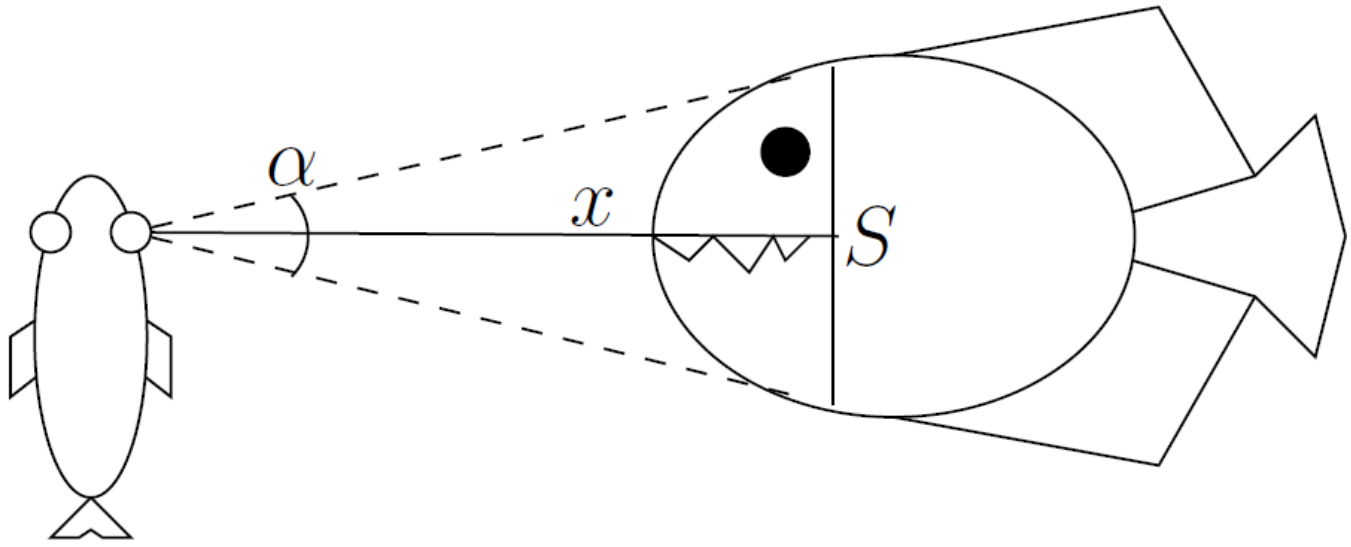
$$\frac{d\alpha}{dt} = \frac{2}{1 + (S/2x)^2}\left(-\frac{S}{2x^2}\right)(-v) = \frac{Sv}{x^2 + S^2/4}$$



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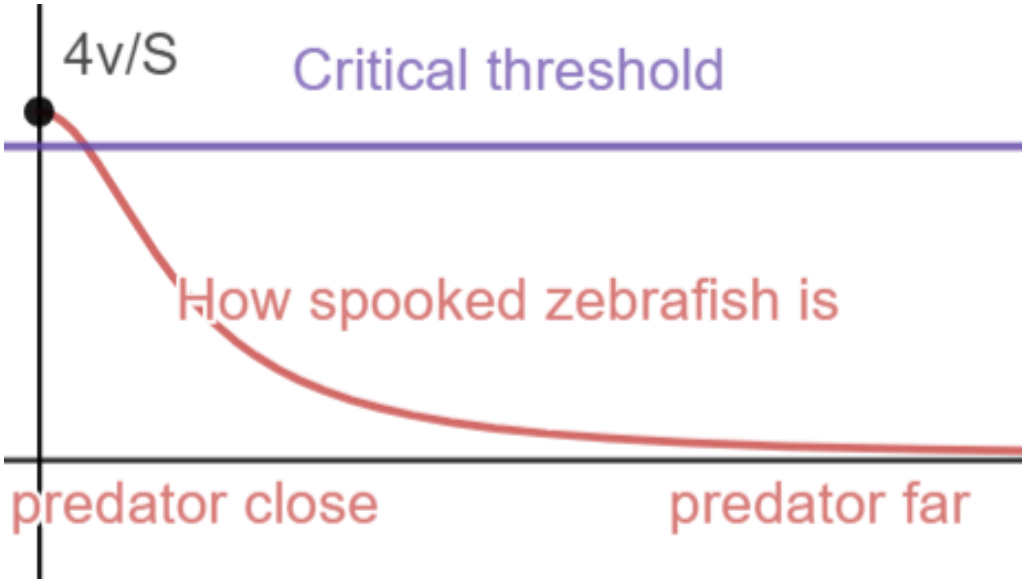
https://www.desmos.com/calculator/aivt2ypu6c

Predator velocity



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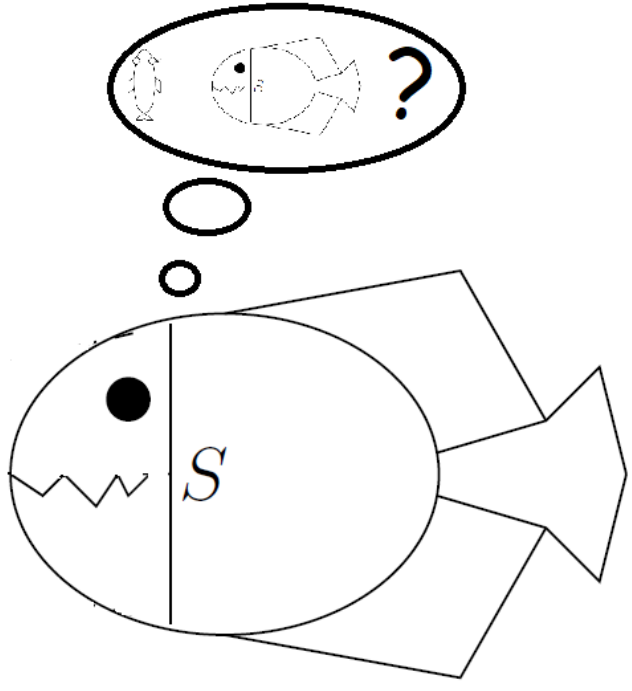

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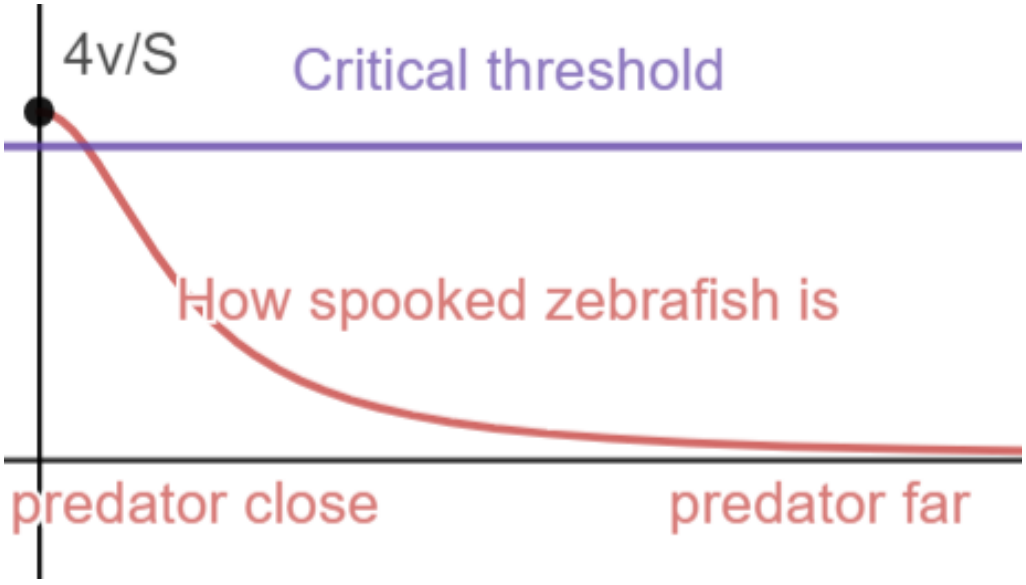
Predator velocity



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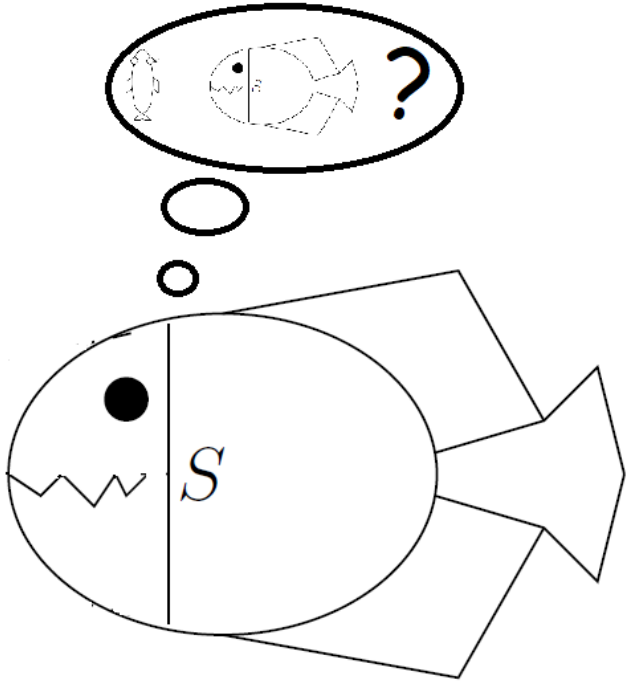
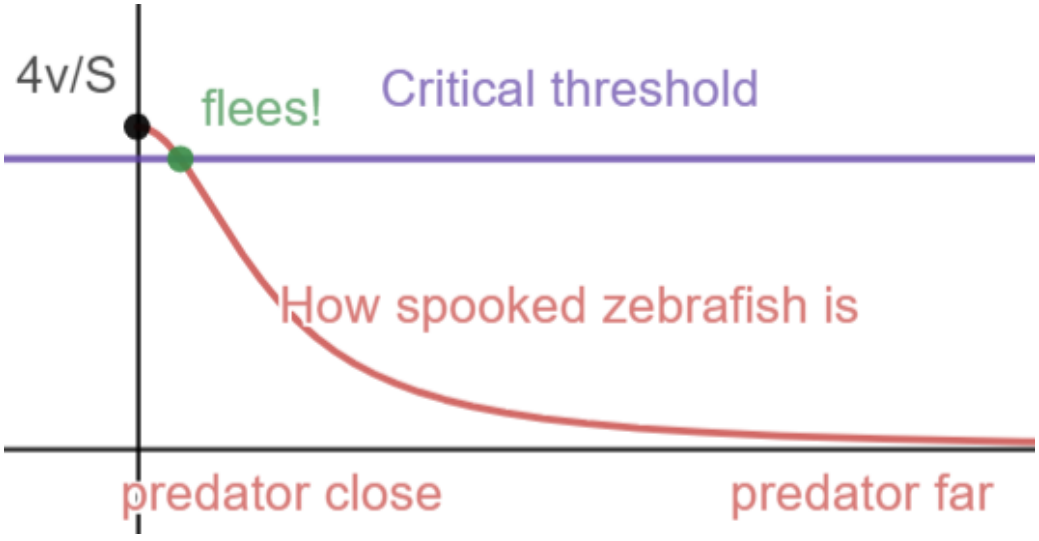


Question: How slowly must a predator of size S approach in order to not spooke the zebrafish? $v < \frac{K_{\text{crit}} S}{4}$

Reaction distance



$$\frac{Sv}{x^2 + S^2/4} = K_{\text{crit}}$$

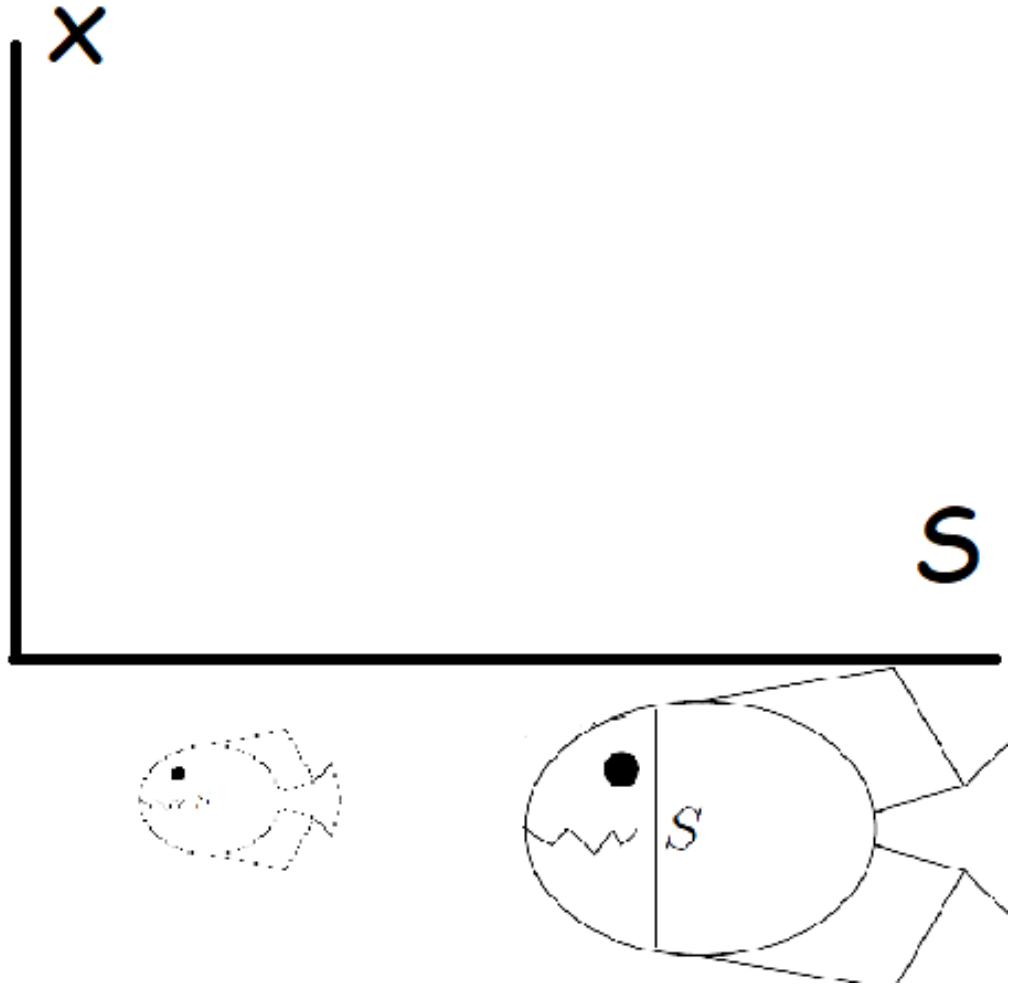
$$\frac{Sv}{K_{\text{crit}}} = x^2 + S^2/4$$

$$x_{\text{react}} = \sqrt{\frac{Sv}{K_{\text{crit}}} - \frac{S^2}{4}}$$

Conclusion: If a predator of size S approaches with velocity v, the zebrafish reacts at distance $x_{\text{react}} = \sqrt{\frac{Sv}{K_{\text{crit}}} - \frac{S^2}{4}}$.

Predator design - Big fish, small fish?

Question: Plot $x_{\text{react}} = \sqrt{\frac{Sv}{K_{\text{crit}}} - \frac{S^2}{4}}$ as a function of S. Assume v and K_{crit} are constants.



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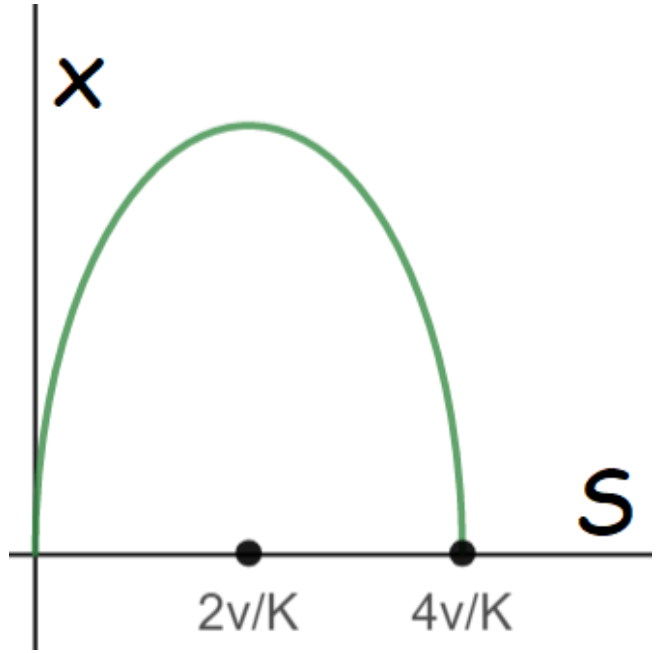

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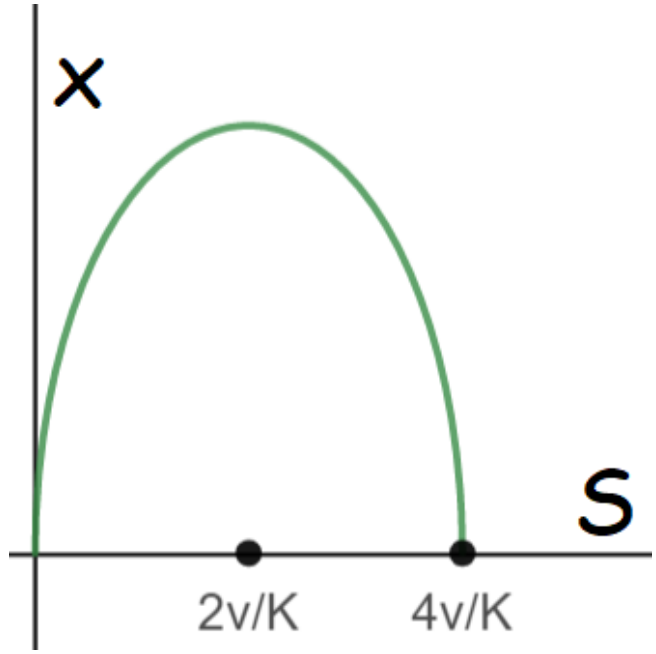

Question: Which predator scares the zebrafish the most? Big, small, or medium?

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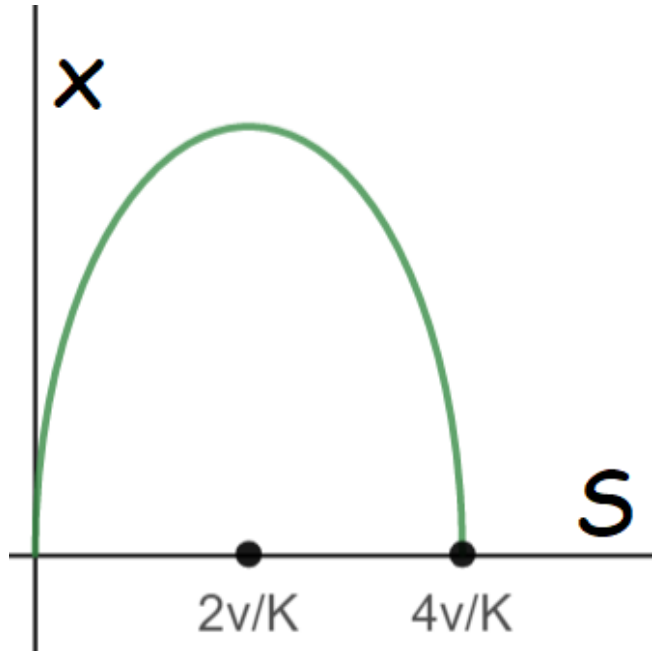

Question: Which predator scares the zebrafish the most? Big, small, or medium? A predator of size $2v/K$.

Thanks for a great semester! GOOD LUCK!